

Ideas sparked by  
*Game Theory & Deep Learning*

Expert Student Talk on CS228  
*Game Theoretical Methodology  
and Technique for Internet Protocols*

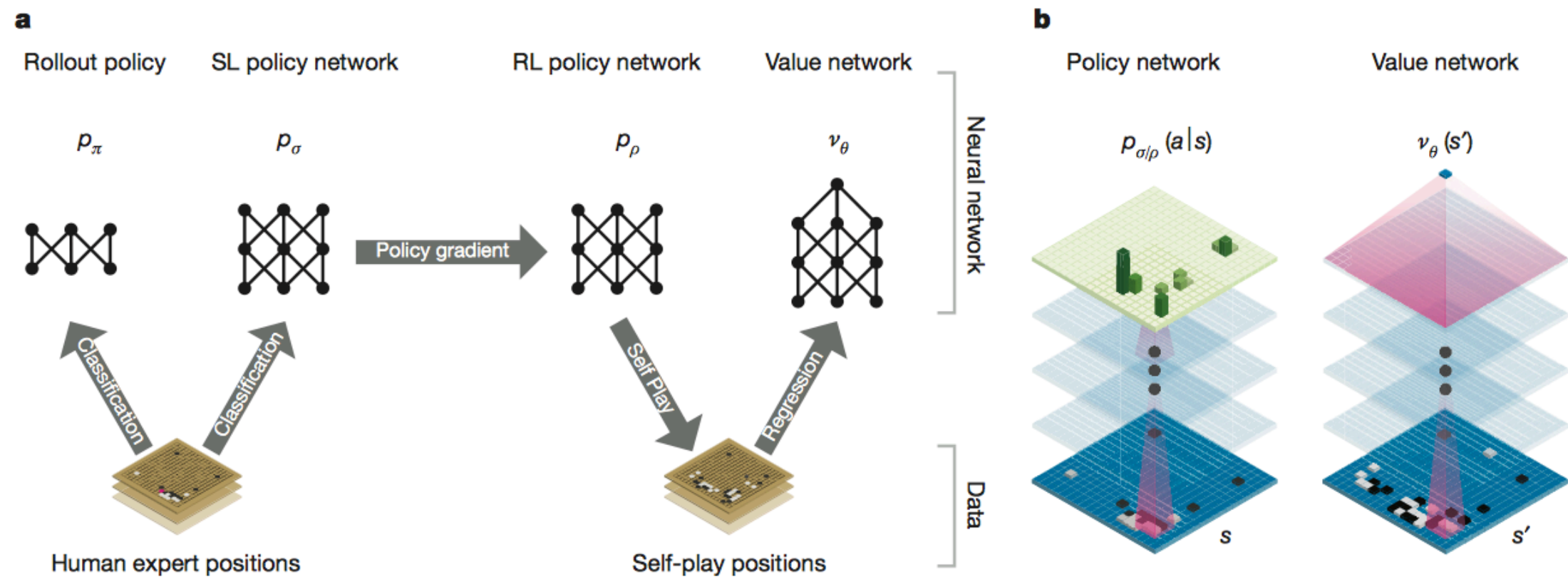
What's happening in AI community?

# Intro: Deep learning in Game



Alpha Go v.s. Lee Sedol, from [youtube.com](https://www.youtube.com/watch?v=Jk2F0jWpW08)

# Intro: Deep learning in Game



Mastering the game of Go with deep neural networks and tree search, from *Nature*



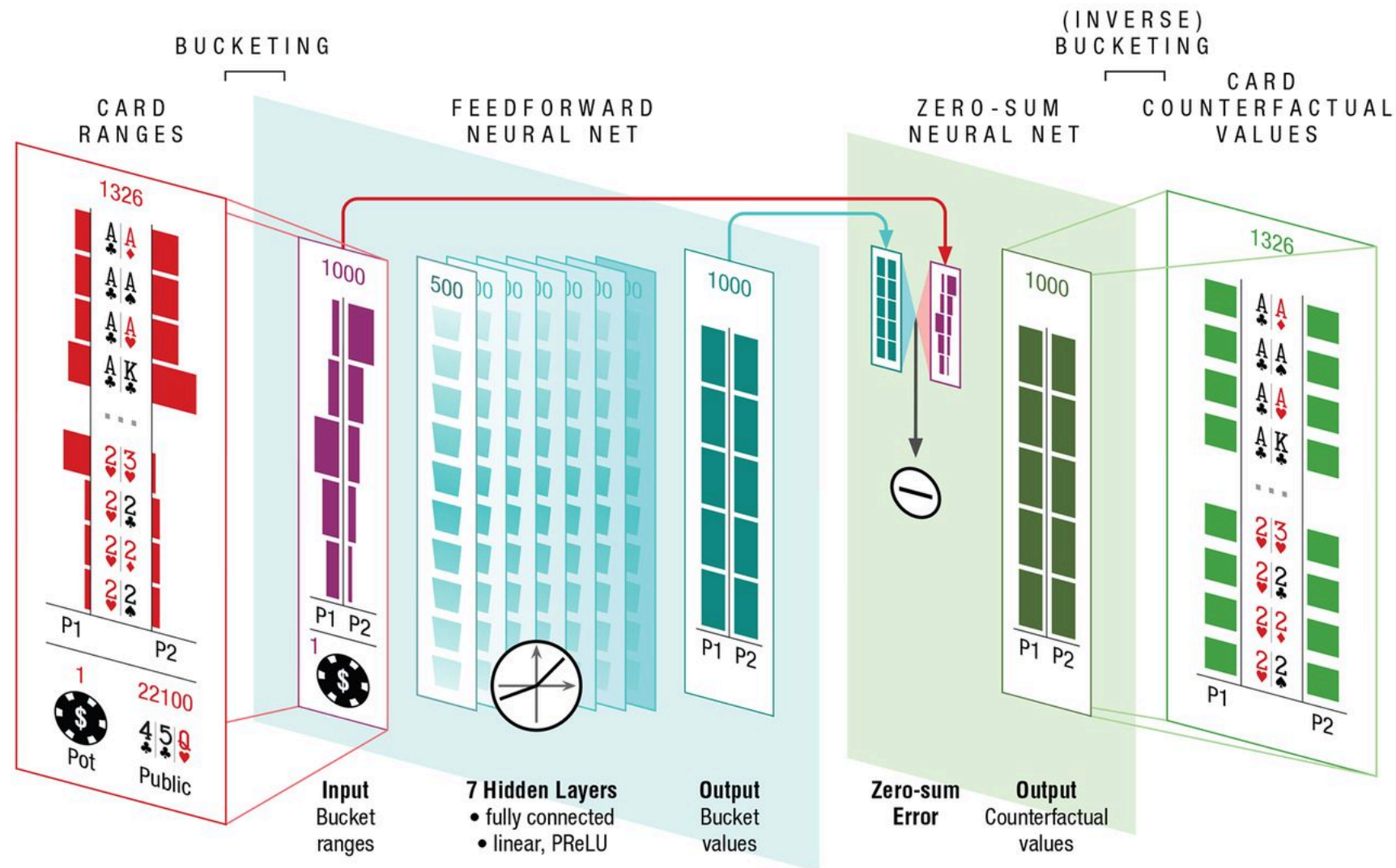
# Intro: Deep learning in Game



The artificial intelligence **Libratus** always knows when to hold 'em and when to fold 'em, from [slate.com](http://slate.com)



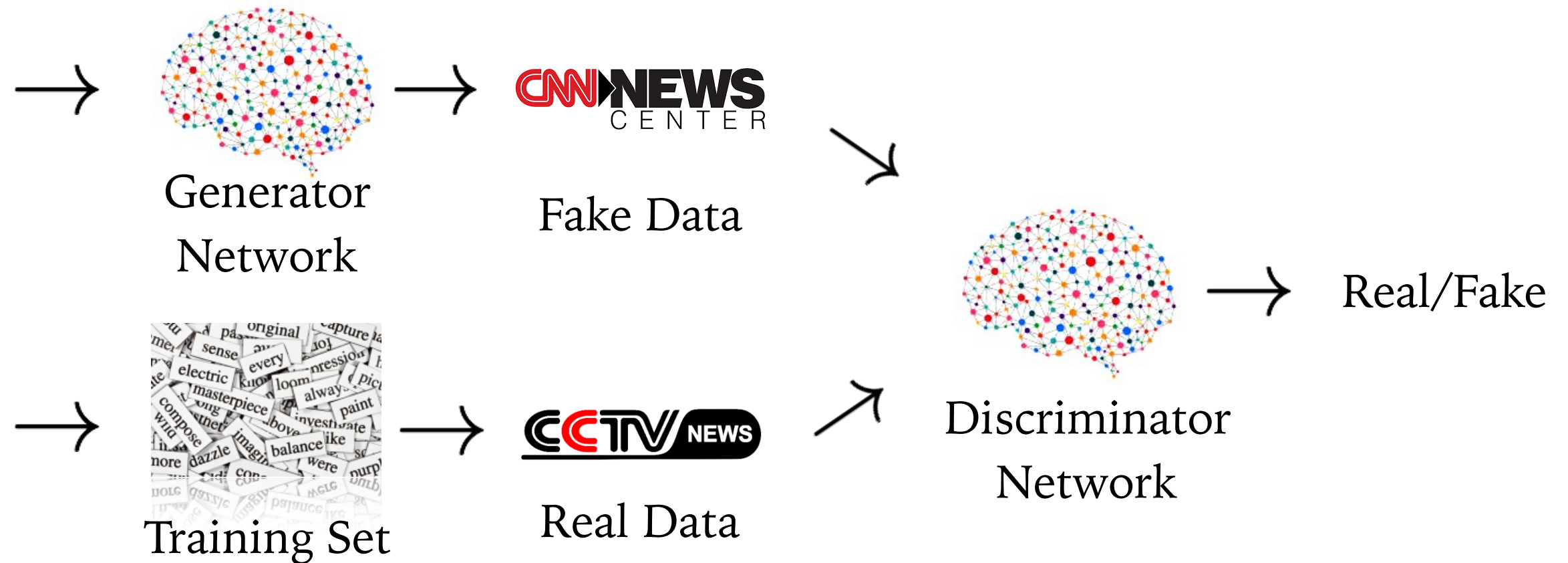
# Intro: Deep learning in Game



DeepStack: Expert-level artificial intelligence in heads-up no-limit poker, from *Science*

# Intro: Game theory in Learning

## Generative Adversarial Nets

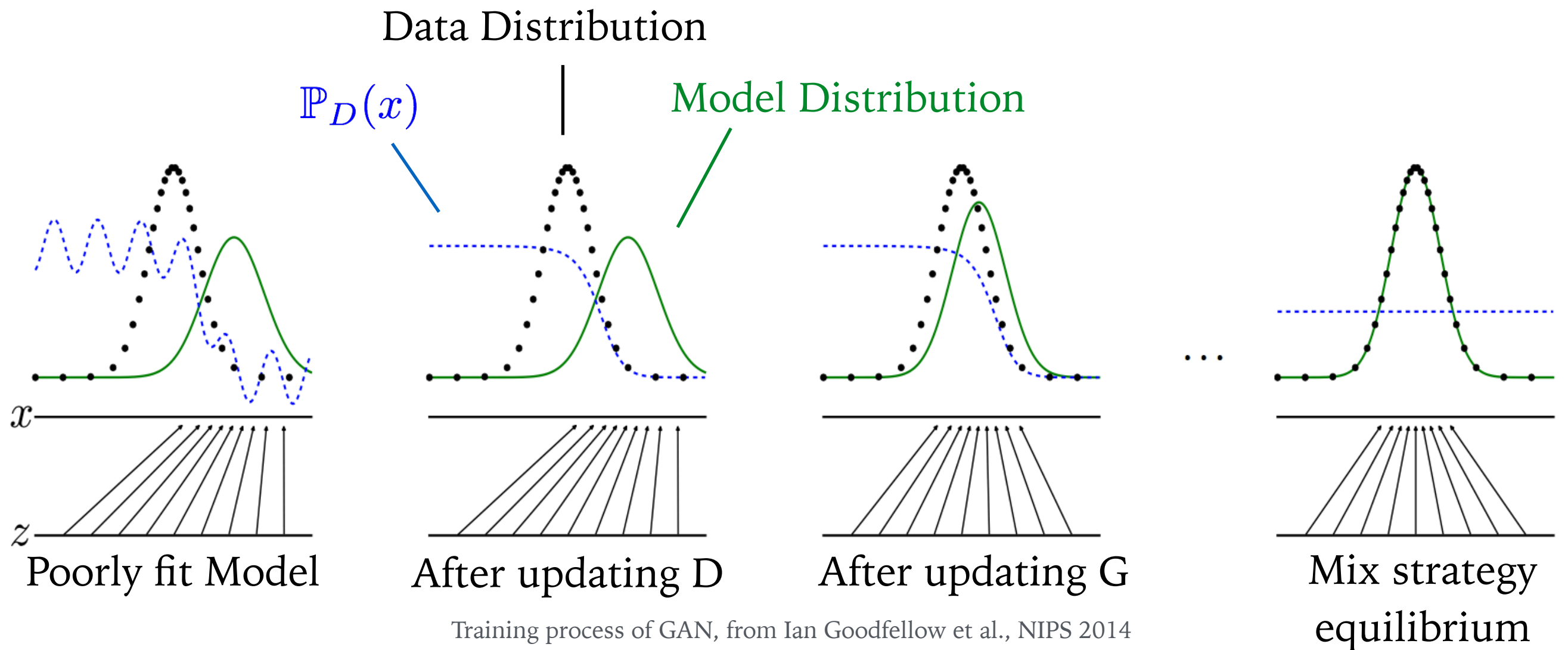


$$\mathcal{L}_{\text{discriminator}}(D) = \mathbb{E}_{\mathbf{x} \sim p}[-\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{x} \sim G}[-\log(1 - D(\mathbf{x}))]$$

$$\mathcal{L}_{\text{generator}}(G) = \mathbb{E}_{\mathbf{x} \sim G}[-\log D(\mathbf{x})] + \mathbb{E}_{\mathbf{x} \sim G}[\log(1 - D(\mathbf{x}))]$$

# Intro: Game theory in Learning

## Generative Adversarial Nets

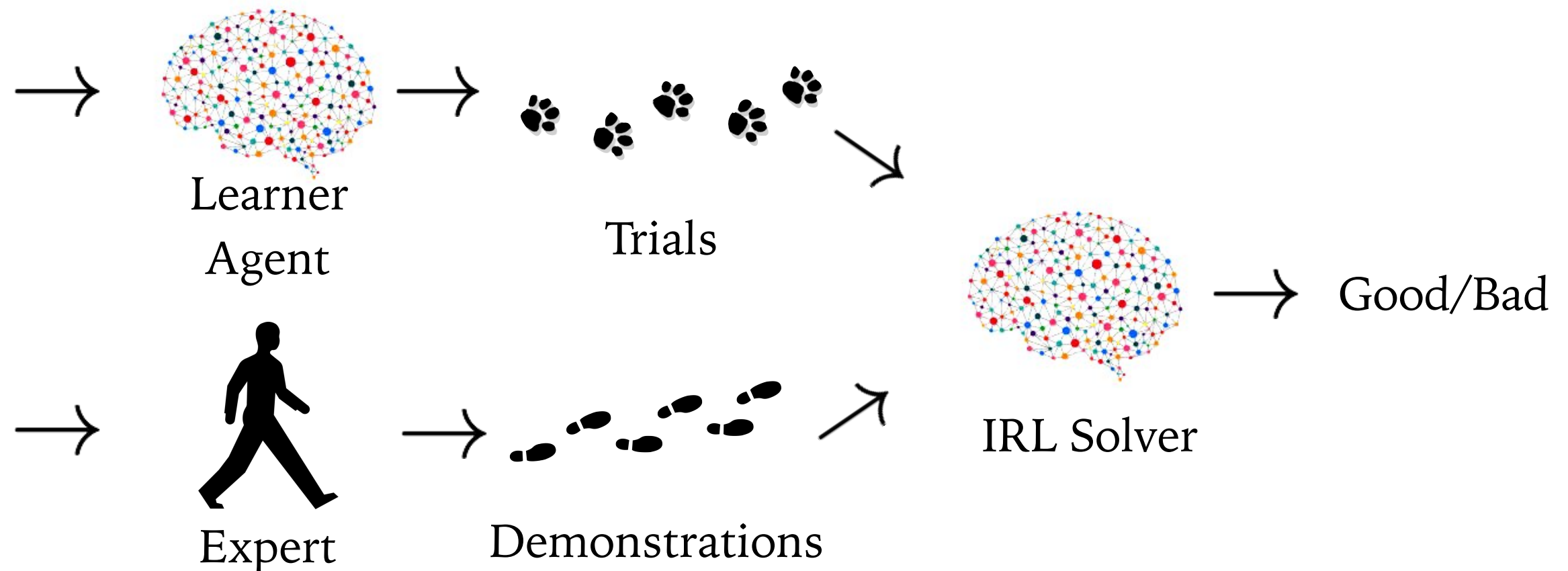


$$\min_{u \in \mathcal{U}} \max_{v \in \mathcal{V}} \mathbb{E}_{x \sim \mathcal{D}_{real}} [\log D_v(x)] + \mathbb{E}_{h \sim \mathcal{D}_h} [\log(1 - D_v(G_u(h)))]$$



# Intro: Game theory in Learning

## Max Entropy Inverse Reinforcement Learning

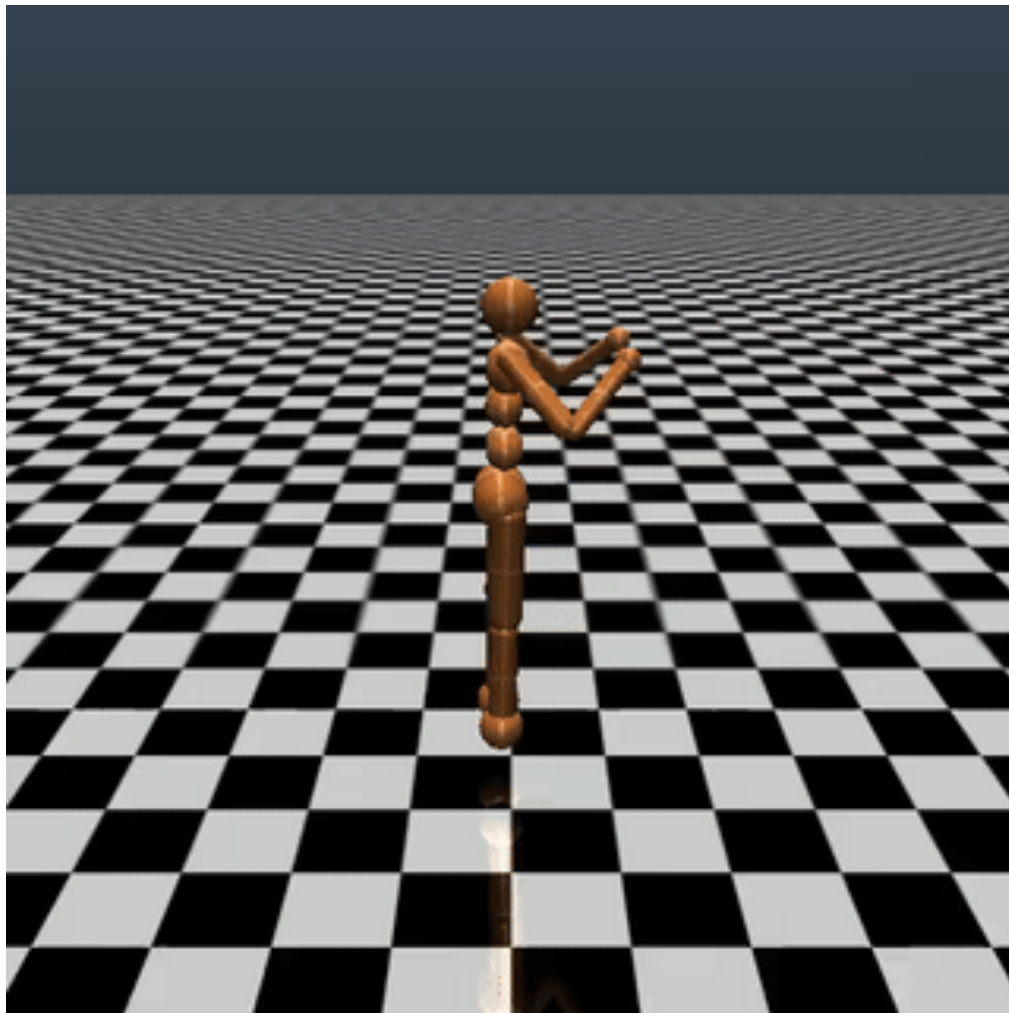


$$\mathcal{L}_{\text{cost}}(\theta) = \mathbb{E}_{\tau \sim \pi^e} [c_{\theta}(\tau)] + \mathbb{E}_{\tau \sim \pi^{\ell}} \left[ \log \left( \frac{\exp(-c_{\theta}(\tau))}{\pi^{\ell}(\tau)} \right) \right]$$

$$\mathcal{L}_{\text{learner}}(\pi^{\ell}) = \mathbb{E}_{\tau \sim \pi^{\ell}} [c_{\theta}(\tau)] + \mathbb{E}_{\tau \sim \pi^{\ell}} [\log \pi^{\ell}(\tau)]$$

# Intro: Game theory in Learning

## Generative Adversarial Imitation Learning



Generative Adversarial Imitation Learning, from Ermon Group, NIPS 2016

Game Theory is elegant but hard to solve.



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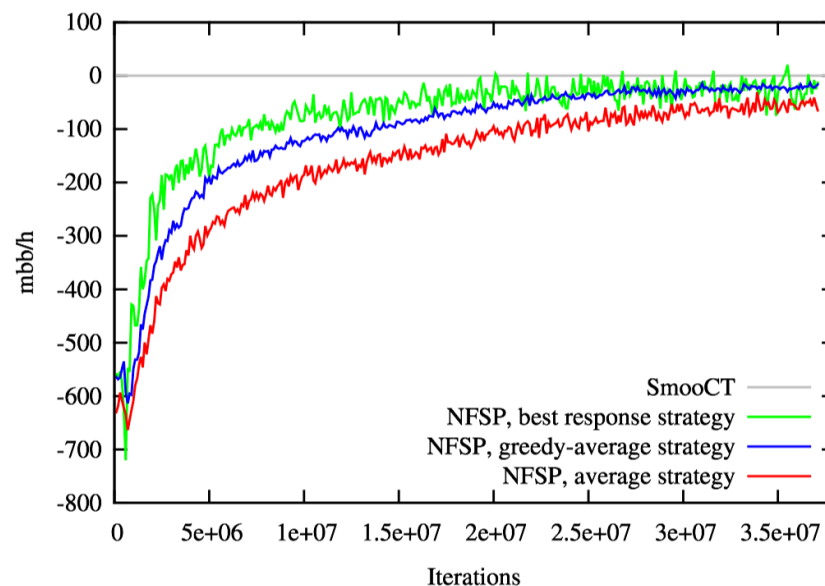
- Plan in Markov Decision Process or POMDP

Game Theory is elegant but hard to solve.

- Plan in Markov Decision Process or POMDP
- Solve Nash Equilibrium with Imperfect Information
  - Counterfactual regret minimization (CFR)
  - Neural Fictitious Self-Play (NFSP)

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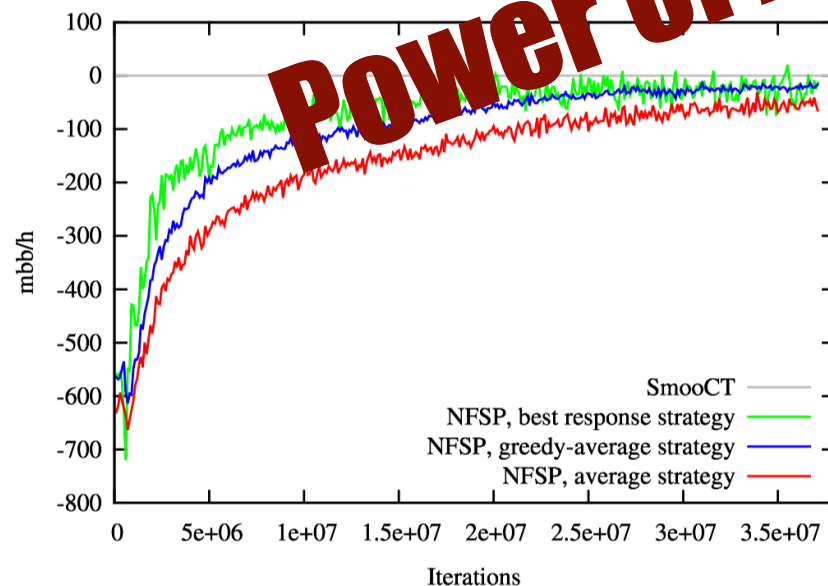
Match-up	Win rate (mbb/h)
escabeche	$-52.1 \pm 8.5$
SmooCT	$-17.4 \pm 9.0$
Hyperborean	$-13.6 \pm 9.2$

Performance of NFSP in Limit Texas Hold'em. David Silver et al.



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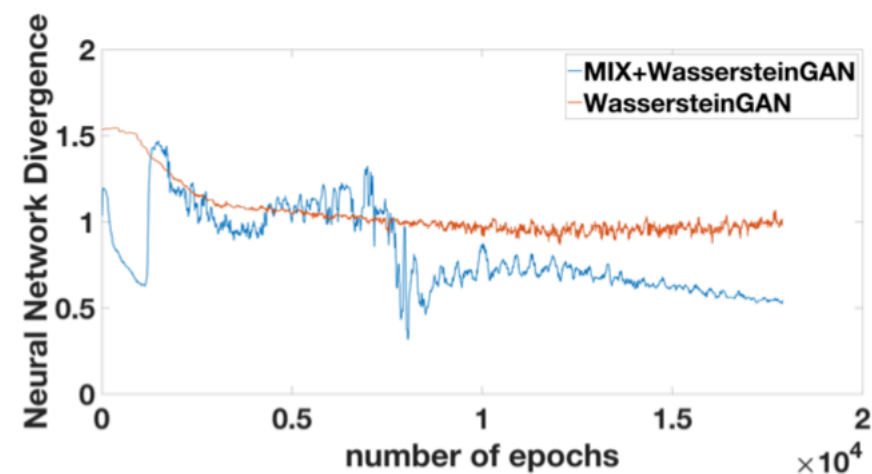
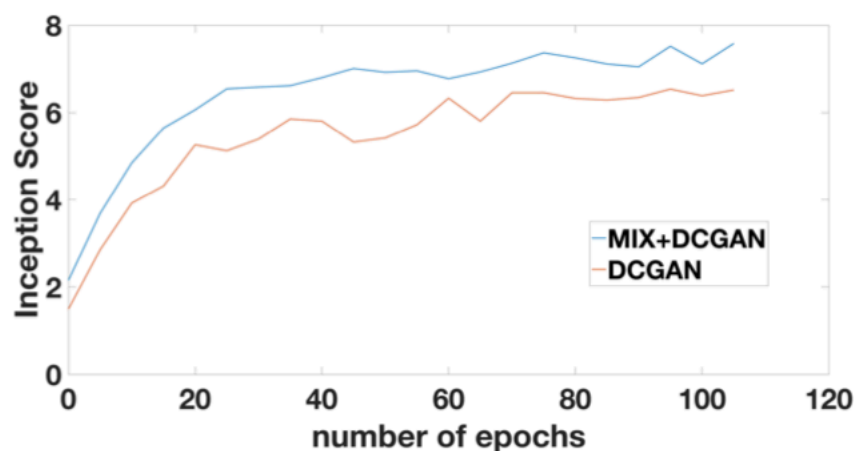


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- Use game theoretical methods to explain and design DL model
- GAN & Imitation Learning

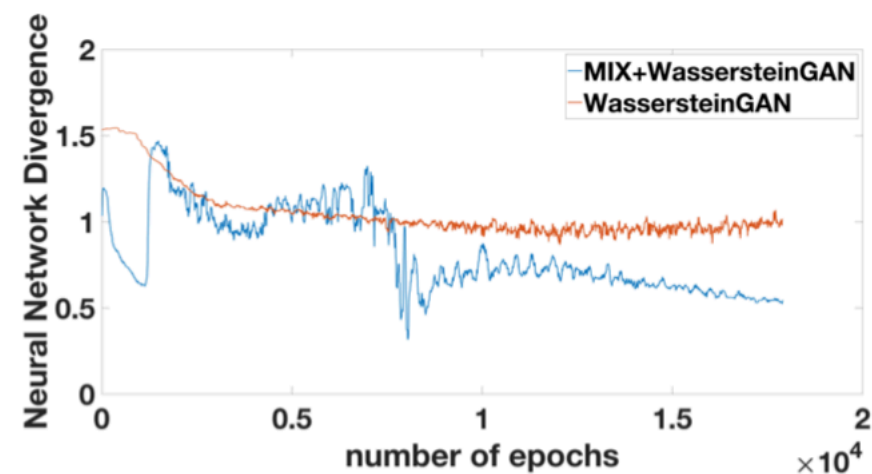
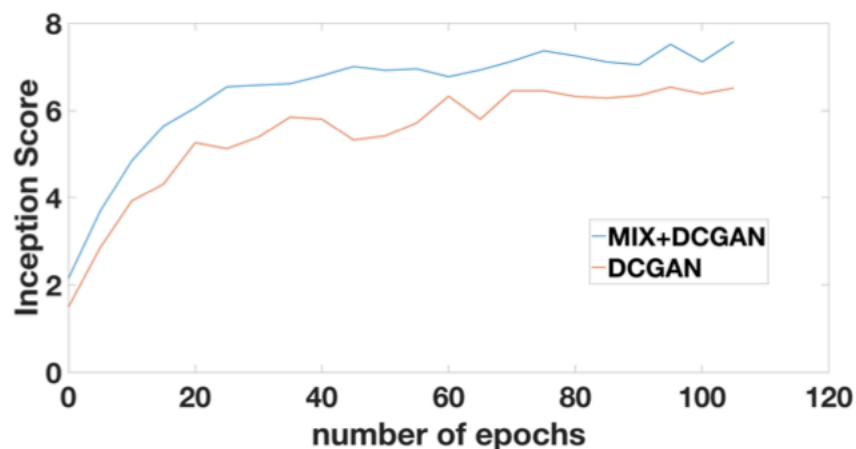
Deep Learning is **pragmatic** but **lacks theoretical guarantee**.

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  - Sanjeev Arora et al. **Generalization and Equilibrium in Generative Adversarial Nets.** [arXiv.org](https://arxiv.org/abs/1703.10717). (2017, March 2)



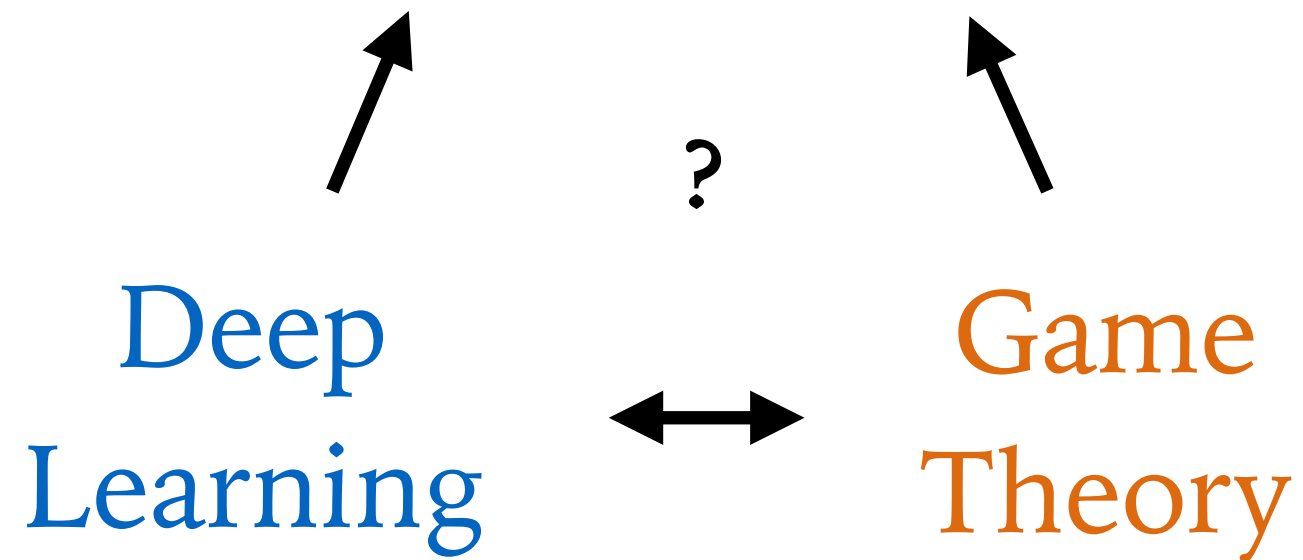
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# Artificial Intelligence



“Humans nowadays completely dominate the planet not because the individual human is **far smarter** and **more nimble-fingered** than the individual chimp or wolf, but because Homo sapiens is the only species on earth **capable of co-operating flexibly in large numbers.**”

*Excerpt From: Yuval Noah Harari.  
Homo Deus: A Brief History of Tomorrow*

# Understanding Agent Cooperation



# Matrix Game Social Dilemmas (MGSD)

# Matrix Game Social Dilemmas (MGSD)

	C	D
C	$R, R$	$S, T$
D	$T, S$	$P, P$

**R** **reward** of mutual cooperation

**P** **punishment** arising from mutual defection

**S** **sucker outcome** obtained by the player who cooperates with a defecting partner

**T** **temptation outcome** achieved by defecting against a cooperator



# Matrix Game Social Dilemmas (MGSD)

	C	D
C	$R, R$	$S, T$
D	$T, S$	$P, P$

*social dilemma inequalities*

- (1)  $R > P$  Mutual cooperation is preferred to mutual defection.
  - (2)  $R > S$  Mutual cooperation is preferred to being exploited by a defector.
  - (3)  $2R > T + S$  This ensures that mutual cooperation is preferred to an equal probability of unilateral cooperation and defection.
- either *greed*:  $T > R$  Exploiting a cooperator is preferred over mutual cooperation
  - or *fear*:  $P > S$  Mutual defection is preferred over being exploited.

# Matrix Game Social Dilemmas (MGSD)

three canonical examples:

Chicken	C	D
C	3, 3	1, 4
D	4, 1	0, 0

Stag Hunt	C	D
C	4, 4	0, 3
D	3, 0	1, 1

Prisoners	C	D
C	3, 3	0, 4
D	4, 0	1, 1

# Matrix Game Social Dilemmas (MGSD)

## Temporal Extension: **Sequential Social Dilemmas**

$$r_i : \mathcal{S} \times \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \mathbb{R} \quad \mathcal{T} : \mathcal{S} \times \mathcal{A}_1 \times \mathcal{A}_2 \rightarrow \Delta(\mathcal{S})$$

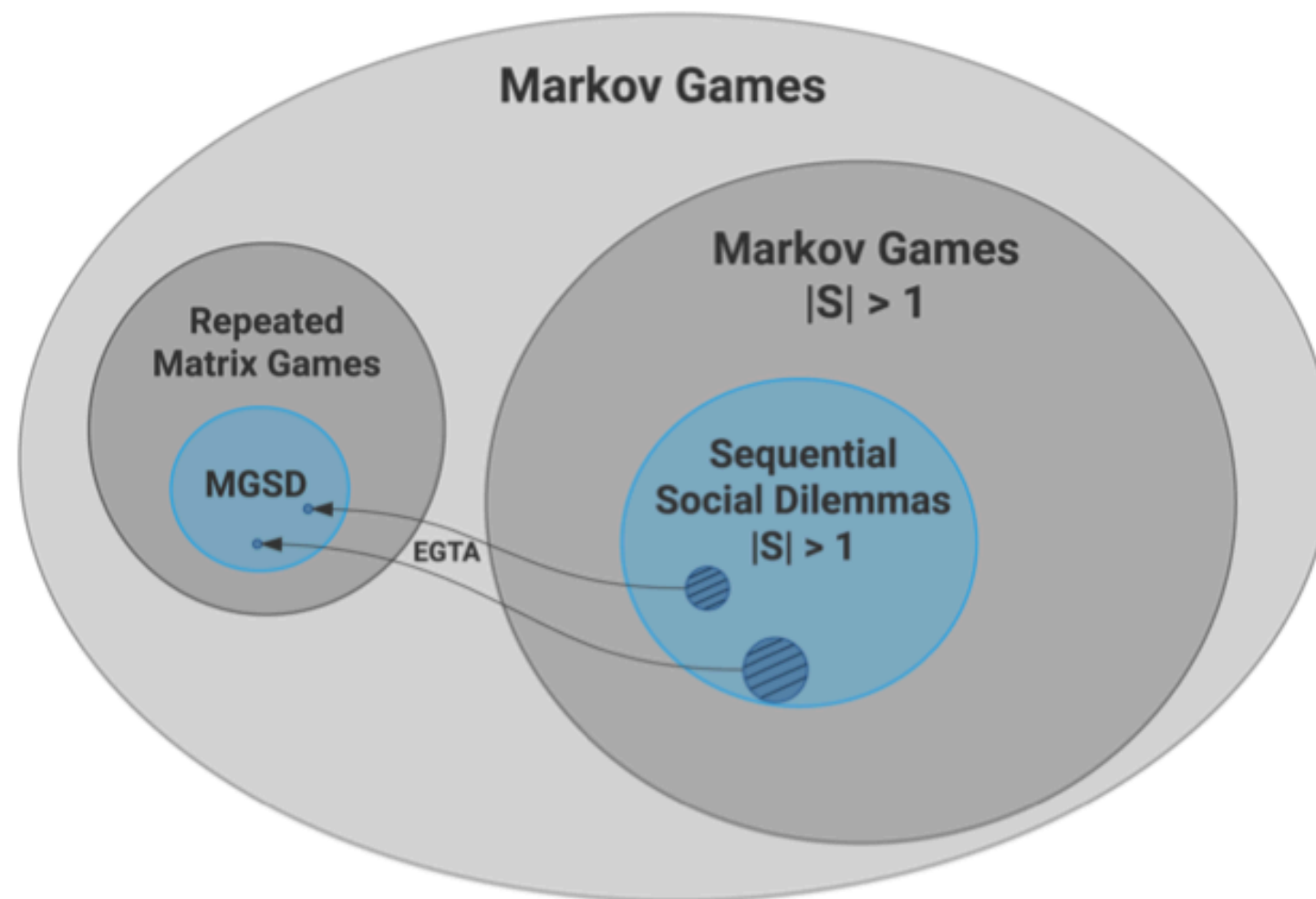
$$(\pi^C \in \Pi^C, \pi^D \in \Pi^D) \Rightarrow (R(s), P(s), S(s), T(s))$$

long-term pay-off:

$$V_i^{\vec{\pi}}(s_0) = \mathbb{E}_{\vec{a}_t \sim \vec{\pi}(O(s_t)), s_{t+1} \sim \mathcal{T}(s_t, \vec{a}_t)} \left[ \sum_{t=0}^{\infty} \gamma^t r_i(s_t, \vec{a}_t) \right]$$

# Matrix Game Social Dilemmas (MGSD)

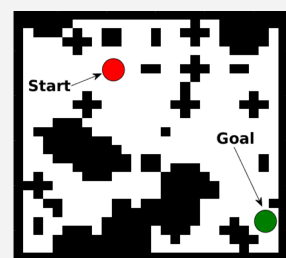
## Temporal Extension: **Sequential Social Dilemmas**



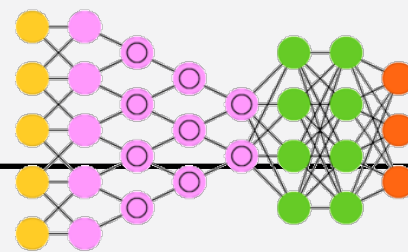
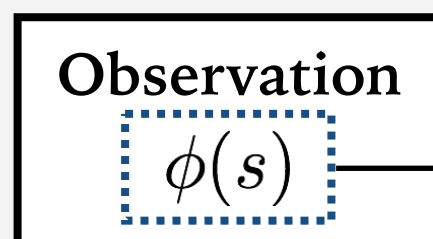
# Sequential Social Dilemmas (SSD)

## Deep Multi-agent Reinforcement Learning

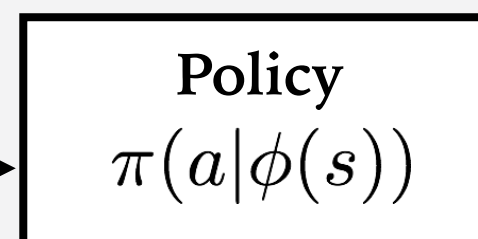
Each agent:



Task  $M$



Deep Q-Net



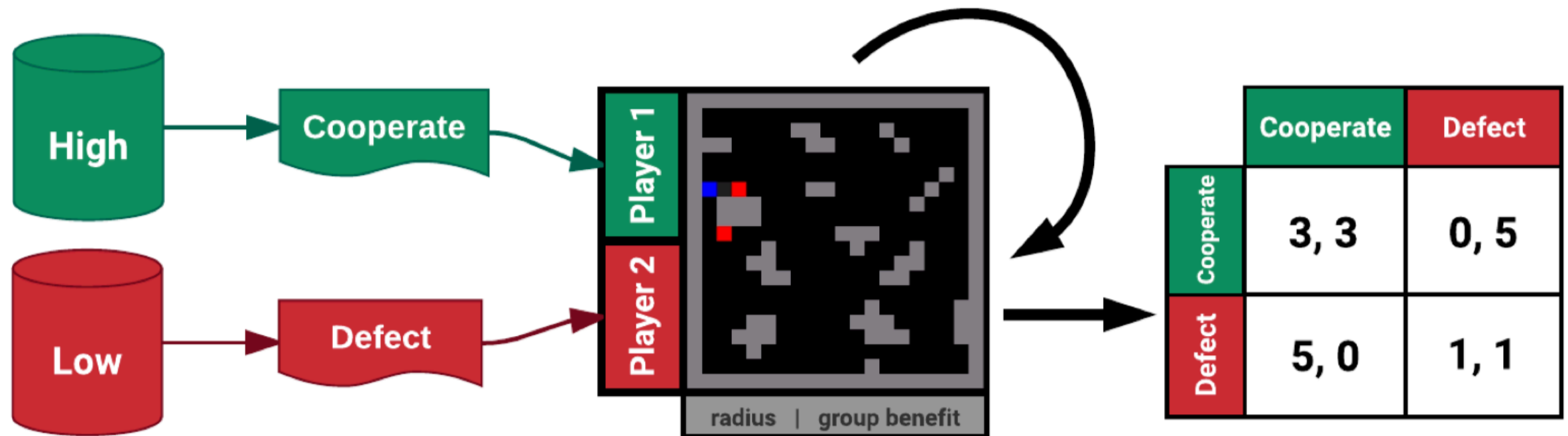
Action  
Probability

$$Q_i(s, a) \leftarrow Q_i(s, a) + \alpha \left[ r_i + \gamma \max_{a' \in \mathcal{A}_i} Q_i(s', a') - Q_i(s, a) \right]$$



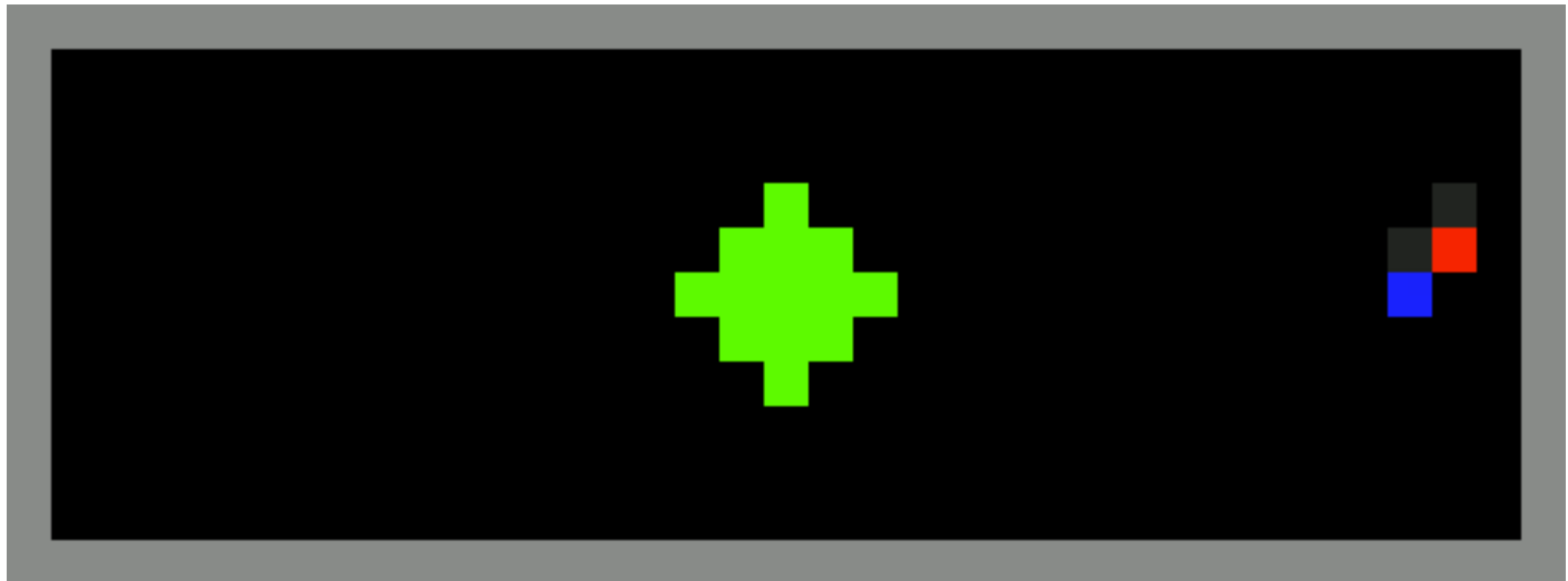
# Sequential Social Dilemmas (SSD)

## Empirical payoff matrices



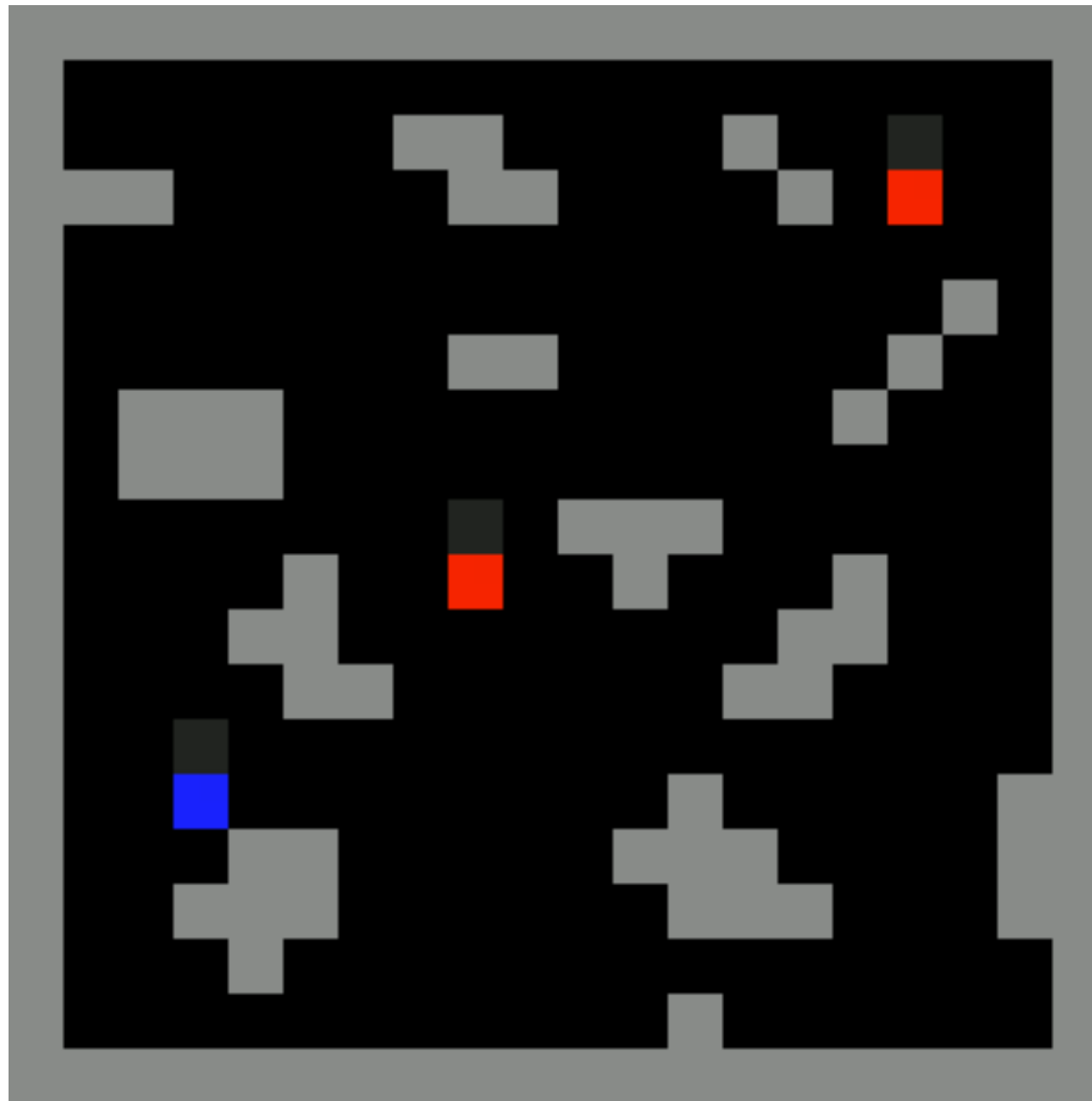
# Sequential Social Dilemmas (SSD)

## Gathering

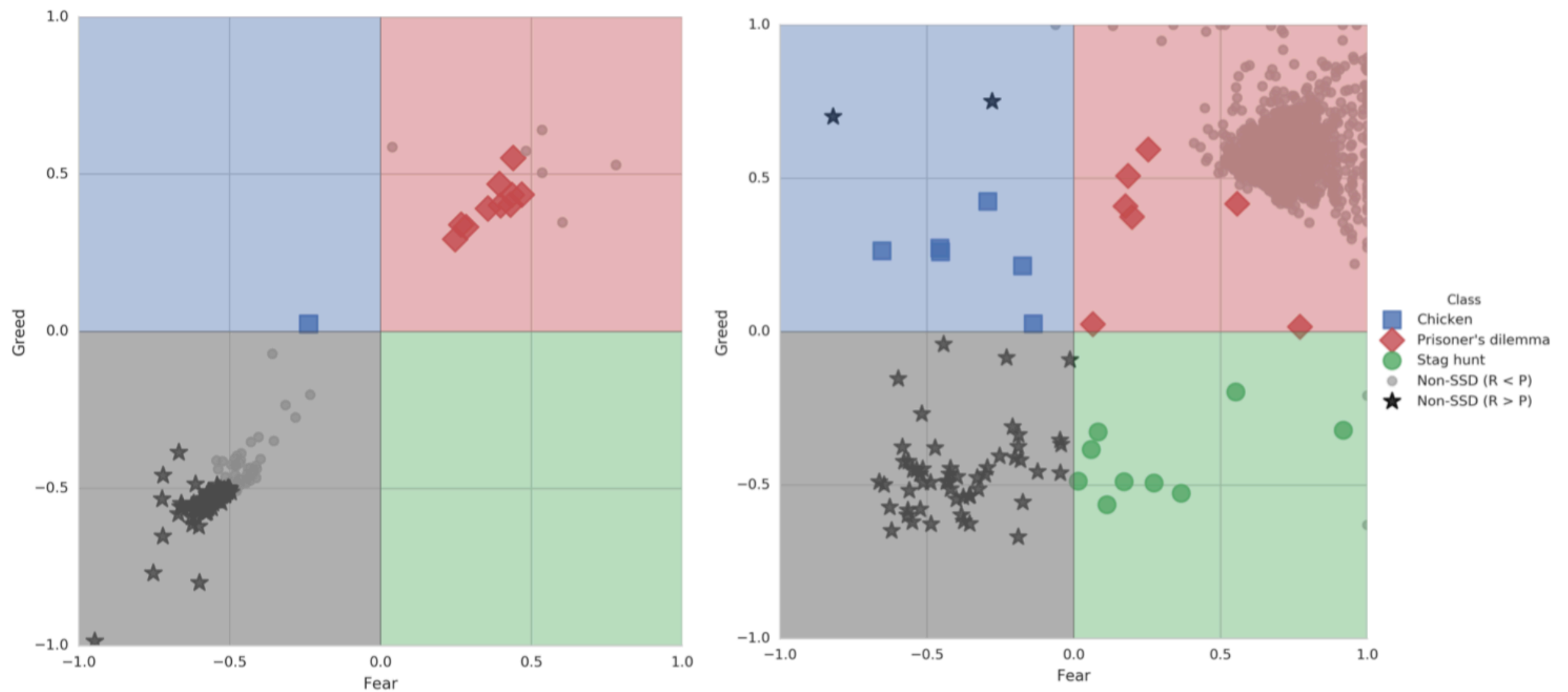


# Sequential Social Dilemmas (SSD)

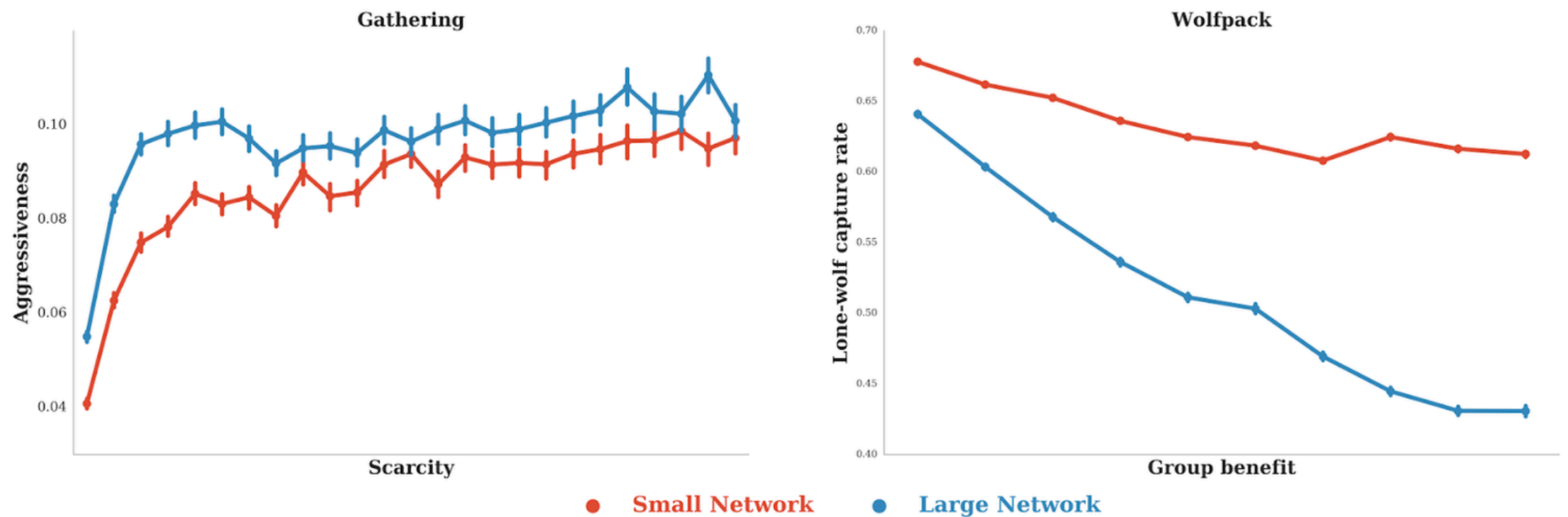
Wolfpack



# Sequential Social Dilemmas (SSD)



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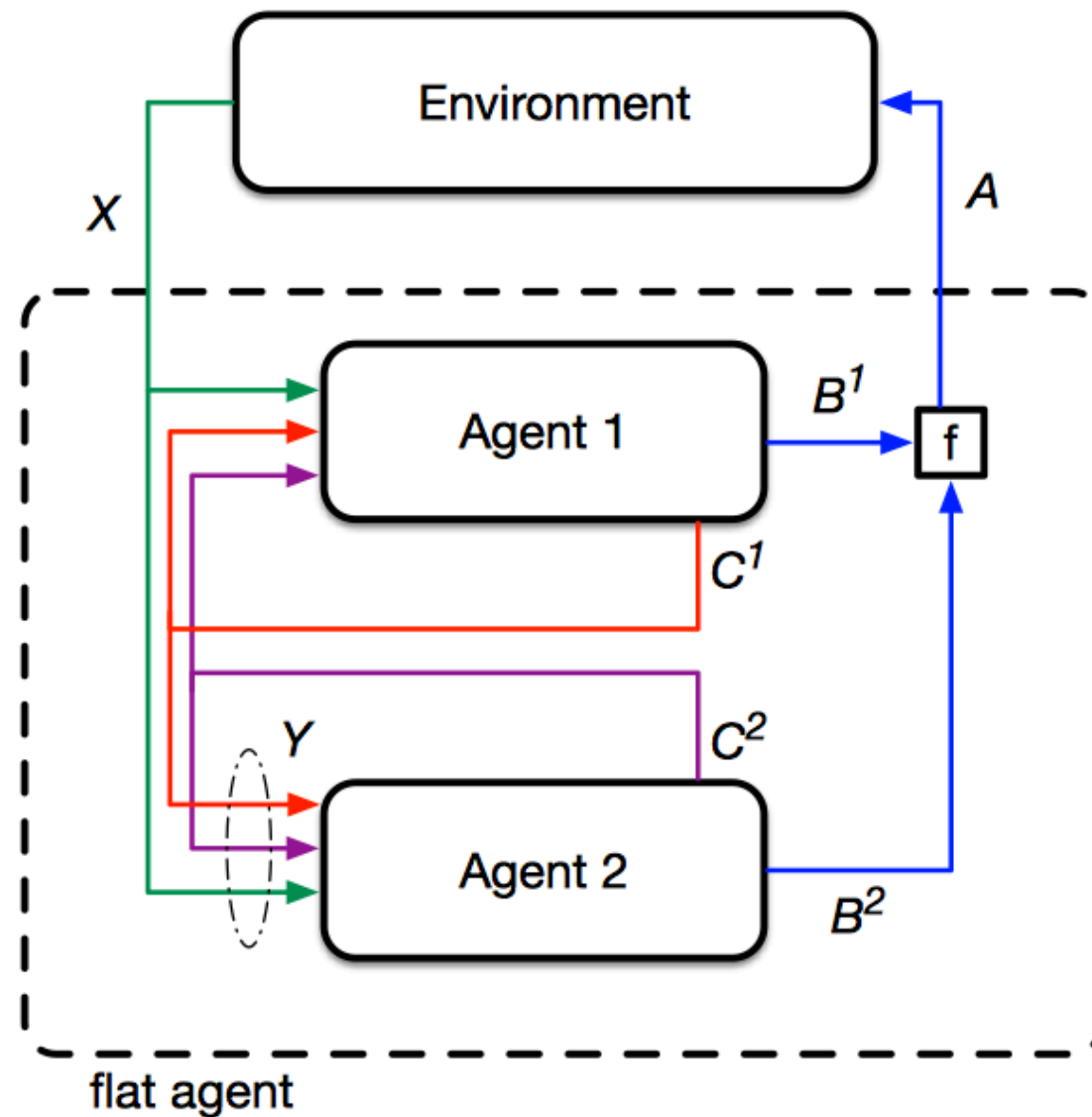
**“Homo Economicus”**

# Specialization: Improve Scalability of RL



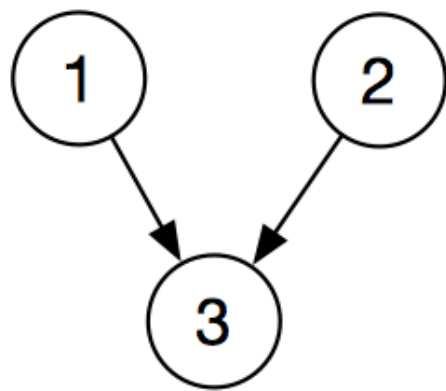
# Scalable Reinforcement Learning

## Separation of Concerns Model

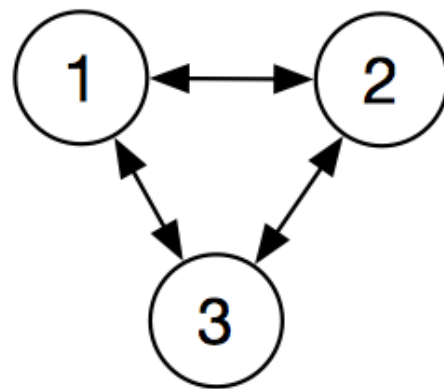


# Scalable Reinforcement Learning

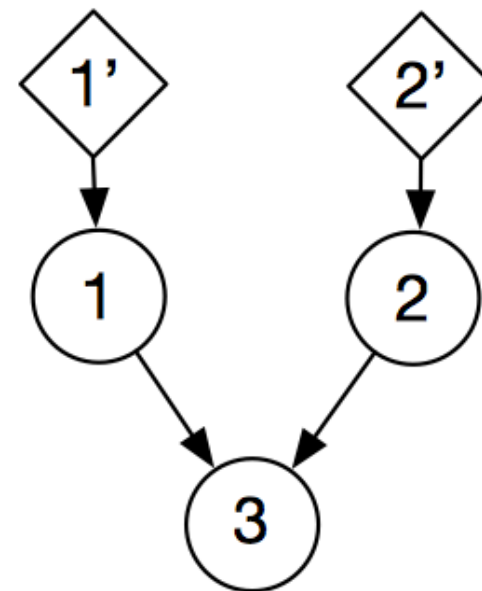
## Convergence



(a)



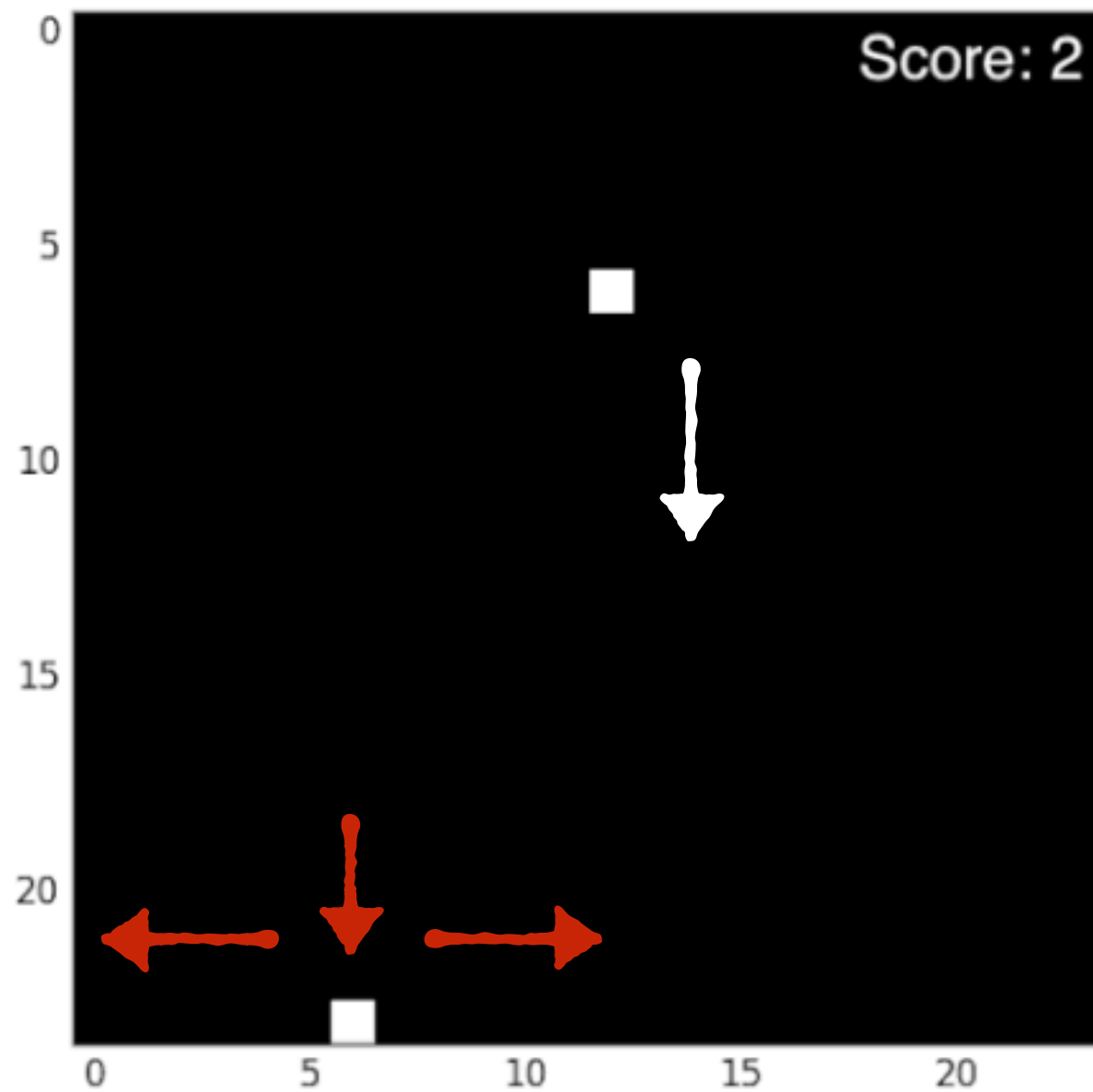
(b)



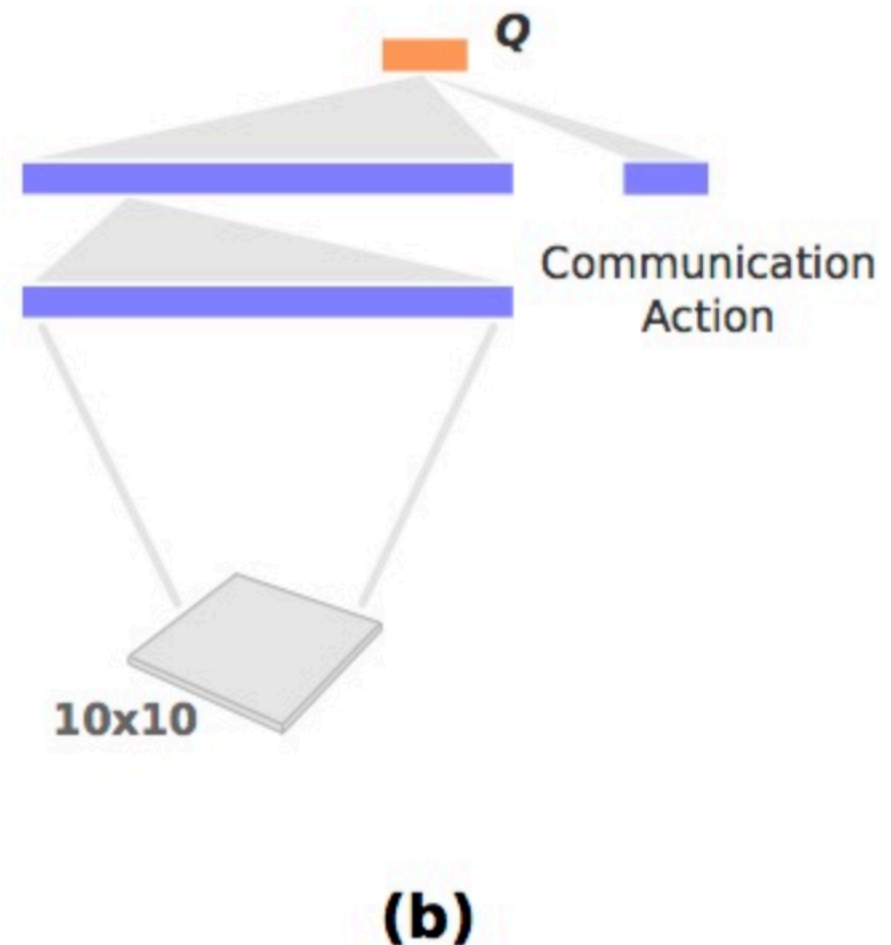
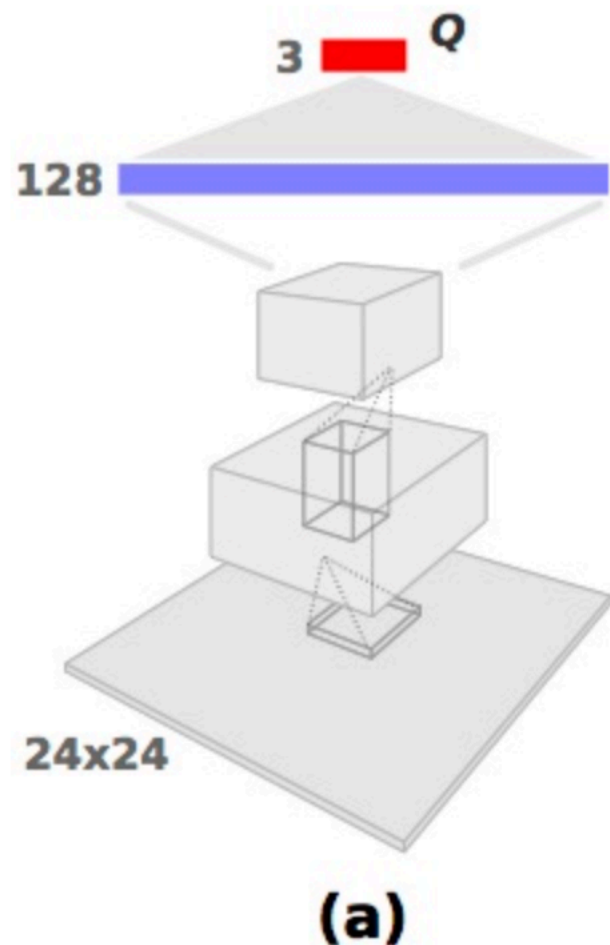
(c)

# Scalable Reinforcement Learning

Catch



# Scalable Reinforcement Learning



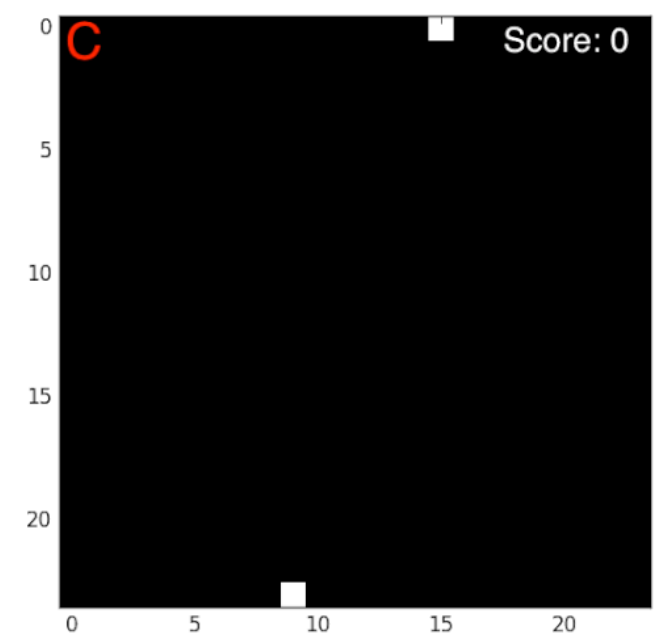
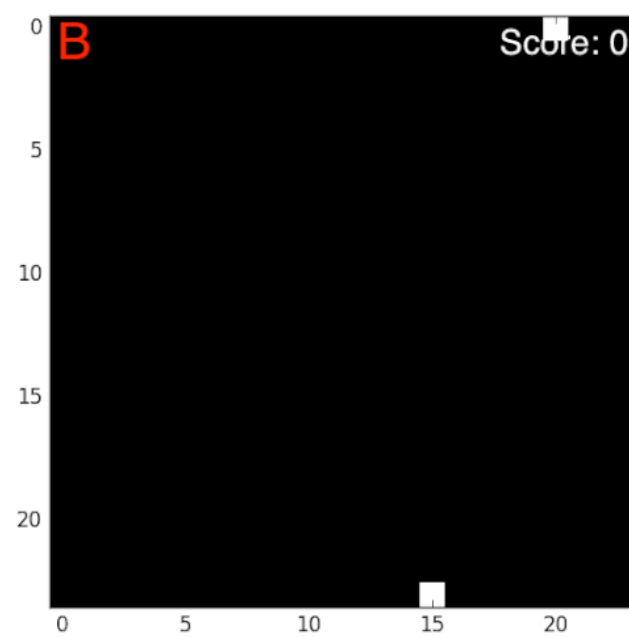
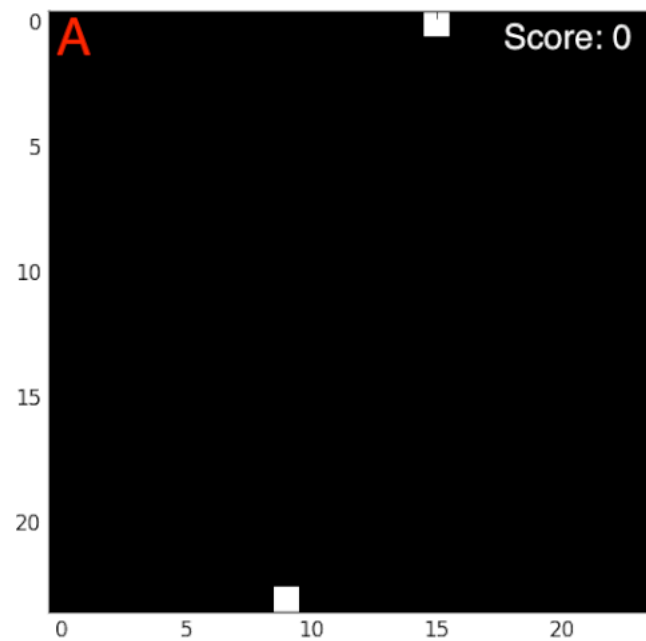
high-level agent:  
high discount factor (adapts slowly)  
accesses to the full screen

low-level agent:  
low discount factor (adapts fast)  
only sees part of the screen



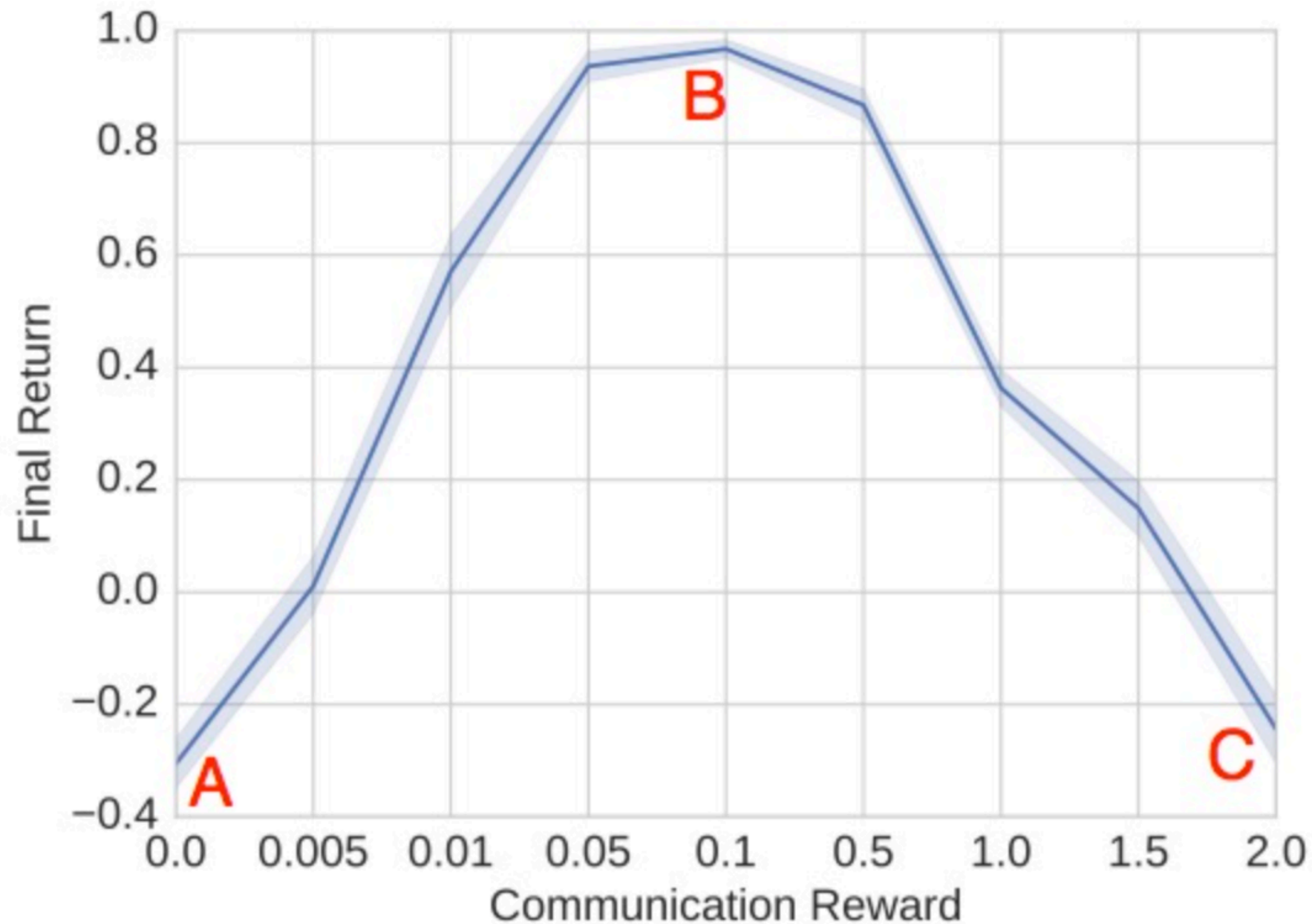
# Scalable Reinforcement Learning

## Catch



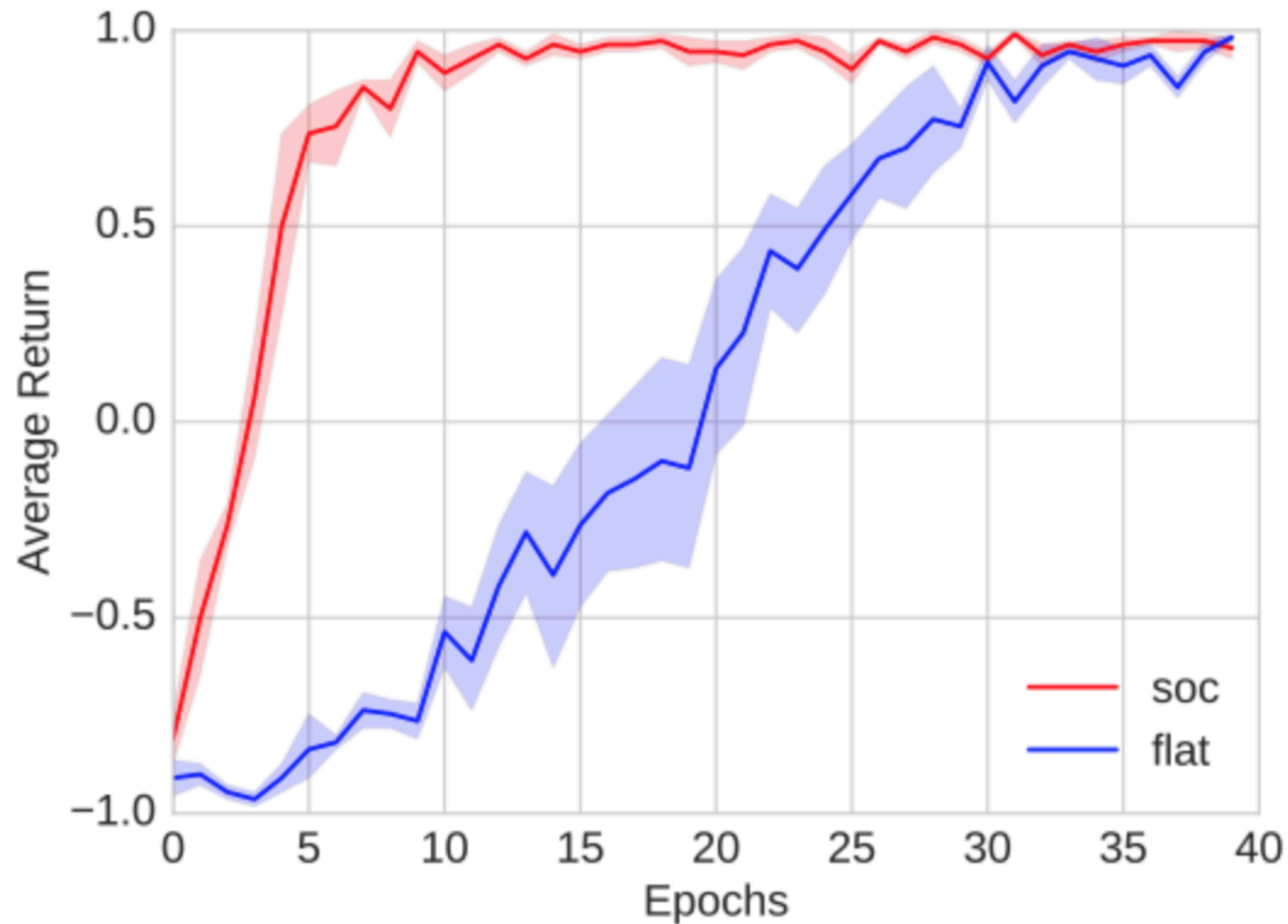
# Scalable Reinforcement Learning

Catch

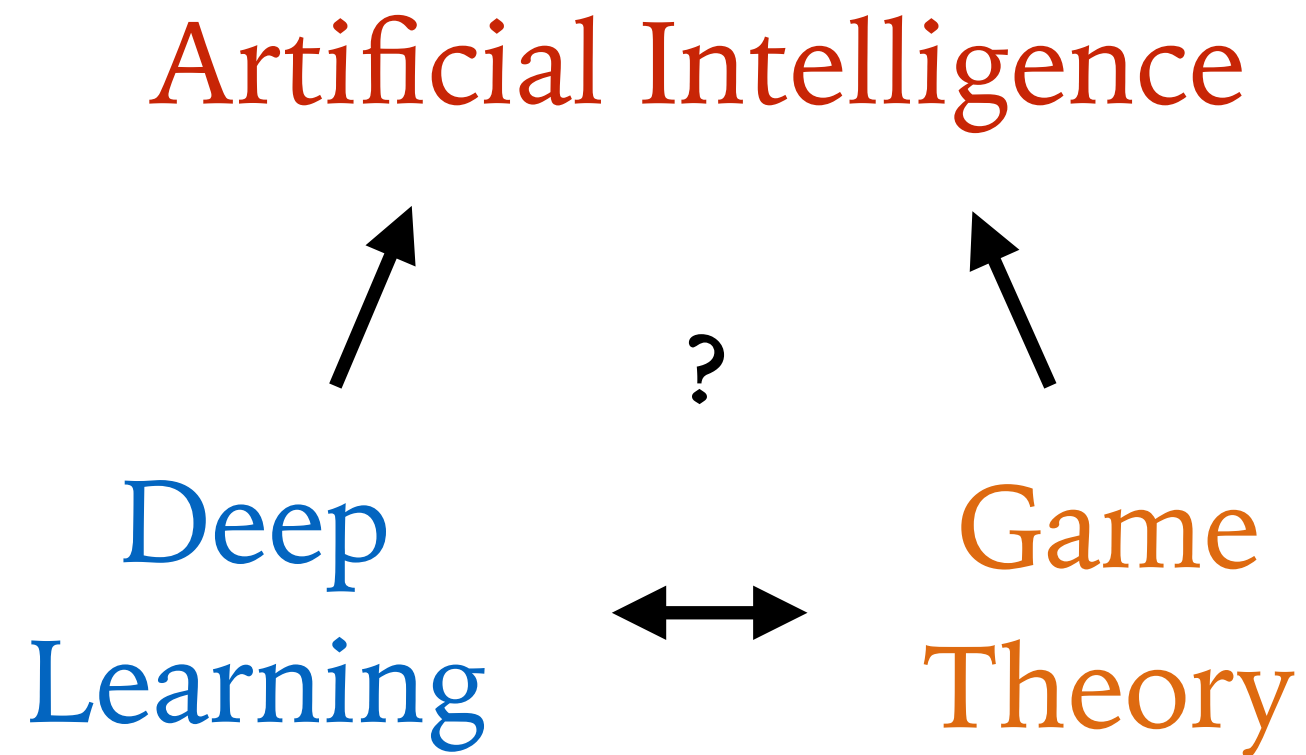


# Scalable Reinforcement Learning

Catch



# My Vision



# My Vision

AI Cooperation, a Cool Future!



Deep  
Learning + Game  
Theory

# References:

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